Inhomogeneous Superconductor in an a.c. Field: Application to the Pseudogap Region

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Abstract

The behavior of an inhomogeneous superconductor in an external a.c. field is studied. General equations describing the a.c. response are formulated. Special attention is paid to the case of a layered conductor containing superconducting "islands". A system of this type displays "pseudogap" properties. The surface impedance Z is evaluated. It is shown that the ReZ ≠ |ImZ| and their difference $\Delta Z \propto \omega^{-1/2}$, ω is the frequency of the a.c. field.

I. **Introduction.**

This paper is concerned with the a.c. response of an inhomogeneous superconductor. The study is a continuation of the analysis presented in our papers [1], [2] . We have developed a model describing a peculiar state

which displays normal d.c. resistivity side by side with superconducting

properties such as a gap structure, diamagnetism, isotope effect, loss of

entropy, etc. This state is inhomogeneous , and the critical temperature is

spatially dependent, so that $T_C=T_C(\mathbf{r})$. At temperatures above $T_C^{res.}$ ($T_C^{res.}$

corresponds to the transition in the dissipationless state; a detailed

description is given in our paper [1]), the sample contains a set of

superconducting regions ("islands") embedded in a normal metallic matrix.

An increase in temperature leads to a decrease in the number of such

clusters and a decrease in their size; the picture persists up to an upper

characteristic temperature T_C^*. The normal matrix provides   a finite normal

resistance whereas the presence of the "islands" and the corresponding

gap parameters results in gap structure, diamagnetic moment, etc.

The size of the superconducting clusters depends on

temperature.The clusters appear at the upper characteristic temperature Tc^*

and their initial size is of order of the coherence length ξ_0 (for the cuprates

ξ_0=15-20A).This lowest limit is determined by the proximity effect.Indeed,the

superconducting state of the region smaller than the coherence length would be

destroyed by the proximity contact with the normal matrix. As was noted

above,each "island" has its own phase.Decrease in temperature below Tc^* leads

to an increase in a number of clusters and, consequently, to an effective increase

in their size.Indeed, the system is inhomogeneous and increase in a number of

"islands" leads to some of them being at short distances (< ξ_0).This provides the

coupling between them (S-N-S Josephson contact) and as a result, the phase

coherence. Eventually, at $T= T_C^{res.}$ We are dealing with formation of

macroscopic region with single phase and macroscopic dissipation less current.

A system of this type will display peculiar a.c. response and, correspondingly, unusual microwave properties. This forms the focus of the present paper. Since the "islands" are embedded in a normal metallic matrix, the proximity effect plays a crucial role (see [1],[2]) which makes the situation different from the usual picture of the phase separation when one deals with a mixture of metallic (or superconducting) and isolating components. Note also that each superconducting "island" is characterized by its own phase, so that the phase coherent state for the whole sample exists only for $T \leq T_c^{res.}$.

Paper [1] contains an evaluation of the density of states (DOS) for such a system. It is shown there that DOS does indeed display a softening in the low energy region , and this is a direct manifestation of gap structure. The diamagnetic response is calculated in [2]; it is shown that, contrary to conventional case, the system is characterized by splitting of the resistive and Meissner transitions.

Generally speaking, there are two sources of inhomogeneity. One of them is a non-uniform distribution of coupling constants (for example, because of a non-uniform doping) which determines the local values of T_C. Another possible source is a non-uniform distribution of pair-breakers (e.g. magnetic impurities; for the D-wave scenario non-magnetic impurities will also act as pair-breakers). Regions with lower concentration of pair-

breakers have higher values of local T_C. Both possibilities were considered in [1] and are studied below.

We think that our approach is directly related to the pseudogap phenomenon which has been observed in the high T_C oxides by a variety of experimental techniques (tunneling [3 - 5], photoemission [6,7], heat capacity [8], isotope effect [9], observation of diamagnetism above T_c^{res} [10 -12], etc). The same is true for the peculiar surface superconducting state observed in the Na-doped WO_3 compound [13], for the Pb-Ag compound described in [14], for granular films, etc.

Diamagnetism above $T_c^{res.}$ has been detected in undoped $La_{2-x}Sr_xCuO_4$ ($T_c^{res.}$ = 18K) by scanning SQUID microscopy [12]. It is remarkable that diamagnetic moment persists up to 80K(!). The authors [12] observe a highly inhomogeneous picture in the pseudogap region and the temperature dependence of the diamagnetic moment is in good agreement with that obtained in [2].

As is known, the pseudogap state in the cuprates has attracted a lot of attention. This state represents a complex phenomenon and can originate from a number of factors, but we think that the presence of spatial inhomogeneites observed experimentally by neutron spectroscopy [15, 16], by STM measurements [17], SQUID STM magnetometry [12] supports our proposition that the inhomogeneous structure of the compounds plays an important role. It is interesting to note also that according to [1, 2], the effects of inhomogeneities are stronger for layered systems than for bulk materials.

As far as a.c. response is concerned, it is known that in normal metals the real and imaginary parts of the surface impedance are almost equal. The situation is entirely different in superconductors (see, e.g., [18]). Below we evaluate the a.c. conductivity and the surface impedance for inhomogeneous superconducting systems. We will focus on the temperature region $T_c^{res.}<T<T_c^*$. As defined above, $T_c^{res.}$ corresponds to the transition from the resistive to the dissipationless state, while the gap structure disappears at T_c^*.

The values of $T_c^{res.}$ and T_c^* are different for various systems. For example, for the underdoped sample of LaSrCuO studied in [12] the values of $T_c^{res.}=18K$ and $T_c^*=80K$ (T_c^* corresponds to disappearance of diamagnetic moment). For the underdoped sample Bi2212 studied by tunneling spectroscopy in [3] $T_c^{res.}=83K$ and $T_c^*=200K$.

The structure of the paper is as follows. Main equations describing the behavior of an inhomogeneous superconductor in the a.c. field are introduced in Section II. Sections III, IV contain an evaluation of the impedance for various types of inhomogenites. The results are discussed in Section V.

II. Main Equations.

A. Inhomogeneous superconductor in the a.c. field

Consider at first, a general case of inhomogeneous superconducting system. To describe such system it is convenient to employ a method of integrated Green's functions developed by Eilenberger [19] and, independently, by A. Larkin and one of the authors [20]. This method was

used to describe thermodynamic and magnetic properties of inhomogeneous systems (see [21, 1, 2]). In this paper we focus on the a.c. response of such superconductor. Consider, at first, the case when the inhomogeneity is caused by a non-uniform distribution of pair-breakers. The case of inhomoheneous coupling will be described below (Sec. IV). The system is described by the equations (see [1], [2]):

$$\alpha\Delta - \beta\omega_n + (D/2)\left(\alpha\frac{\partial^2\beta}{\partial r^2} - \beta\frac{\partial^2\alpha}{\partial r^2}\right) = \alpha\beta\Gamma \qquad (1')$$

$$\alpha^2 + |\beta|^2 = 1 \qquad (1'')$$

$$\Delta = 2\pi T|\lambda|\sum_{\omega_n>0}\beta(\omega_n) \qquad (1''')$$

Here α and β are the usual and the pairing [22] Greens functions integrated over energy, D is the diffusion coefficient (we consider a "dirty" case), $\Gamma \equiv \Gamma(\mathbf{r}) = \tau_s^{-1}$, τ_s is a spin-flip relaxation time [23], Δ is the order parameter. We are using the method of thermodynamic Green's functions (see e.g. [24]), so that $\omega_n = (2n+1)\pi T$.

We focus on the high temperature region $T > T_C^{res}$, although $T < T_C^*$. The order parameter is small so that $\Delta < \Gamma + 2\pi T$. As a result, in a first approximation, the system (1) has a form:

$$\beta_o = \hat{L}^{-1}\Delta \qquad (2')$$

$$\alpha_o = 1 - \beta_o^2/2 \qquad (2'')$$

$$\Delta = 2\pi T|\lambda|\sum_{\omega_n>0}\hat{L}^{-1}\left[\Delta - (\beta_o/2)(\omega_n\beta_o^2 - (D/2)\partial^2\beta_o^2/\partial r^2)\right] \qquad (2''')$$

Here

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$$\hat{L} = \omega_n + \Gamma(\mathbf{r}) - (D/2)\partial^2 / \partial \mathbf{r}^2 \tag{3}$$

Consider an external a.c. field

$$\mathbf{A}(\mathbf{r},\tau) = \exp(-i\omega_0\tau)\mathbf{A}(\mathbf{r}); \quad \varphi(\mathbf{r},\tau) = \exp(-i\omega_0\tau)\varphi(\mathbf{r}) \tag{4}$$

(**A** and φ are the vector and scalar potentials, ω_0 is the frequency, τ is an imaginary time). As was shown by A. Larkin and one of the authors (see [25]), in the presence of such field one can write (in the matrix representation):

$$\hat{G} = \hat{G}_0 + \exp(-i\omega_0\tau)\hat{G}_1(\tau - \tau'); \quad \hat{\Delta} = \hat{\Delta}_0 + \hat{\Delta}_1\exp(-i\omega_0\tau)$$

$$\hat{G}_0 = \begin{pmatrix} \alpha & -i\beta \\ i\beta^* & \alpha \end{pmatrix}, \quad \hat{G}_1 = \begin{pmatrix} g_1 & f_1 \\ -f_2 & g_2 \end{pmatrix}; \quad \hat{\Delta}_0 = i\hat{\sigma}_y\Delta_o; \quad \hat{\Delta}_1 = \begin{pmatrix} 0 & \Delta_1 \\ -\Delta_2 & 0 \end{pmatrix} \tag{5}$$

The quantities f_i, g_i are satisfied by the relations which can be obtained from equation for $\hat{G}_1$ (see Appendix) and normalization conditions, and have a form:

$$g_1 = -i(\beta f_1 + \beta_+ f_2)A_+^{-1}; \quad g_2 = i(\beta_+ f_1 + \beta f_2)A_+^{-1}$$

$$f_1 = i(\beta g_1 + \beta_+ g_2)A_-^{-1}; \quad f_2 = -i(\beta_+ g_1 + \beta g_2)A_-^{-1} \tag{6}$$

$$\alpha_+ = \alpha(\omega_n + \omega_0); \quad \beta_+ = \beta(\omega_n + \omega_0); \quad A_\pm = \alpha_+ \pm \alpha$$

In order to describe the a.c. response, we need to evaluate the current density; the expression for **j** can be written in the form [25]:

$$j = \pi\sigma\left\{ T\sum_{\omega_n} A_1(1 - \alpha\alpha_+ - \beta\beta_+) - \frac{1}{2e}T\sum_{\omega_n}(f_1 - f_2)\frac{\partial\beta}{\partial\mathbf{r}} + \right.$$

$$+\frac{1}{2e}T\sum_{\omega_n}\beta_+\frac{\partial}{\partial\mathbf{r}}(f_1 - f_2) + \frac{i}{2e}T\sum_{\omega_n}\alpha_+\frac{\partial}{\partial\mathbf{r}}(g_1 + g_2) + \tag{7}$$

$$\left. +\frac{i}{2e}T\sum_{\omega_n}(g_1 + g_2)\frac{\partial\alpha}{\partial\mathbf{r}} \right\}$$

σ is the d.c. conductivity

Performing an analytical continuation (see Appendix) and using Eq. (6) we obtain:

$$j = i\sigma\omega A_1 - \sigma\frac{\partial\varphi}{\partial\mathbf{r}} - \sigma A_1\frac{\Delta_0^2}{\pi T}\psi'(0.5 + \Gamma_\infty/2\pi T) - $$

$$-\frac{i\sigma}{2e\pi T}\psi'(0.5 + \Gamma_\infty/2\pi T)\left(\Delta_0\frac{\partial\Delta_1}{\partial\mathbf{r}} - \Delta_1\frac{\partial\Delta_0}{\partial\mathbf{r}}\right)$$

(8)

We used also the equation for the scalar potential [25]

$$2e\varphi + i\pi T\sum_n(g_1 + g_2) = 0$$

(9)

Finally, one can write out the equation for the order parameter which follows from Eq. (2''')

$$\left|\ln\left(\frac{T_c^\circ}{T}\right) + \psi(1/2) - \psi\left(1/2 + \frac{\Gamma(\mathbf{r})}{2\pi T} - \frac{i\omega}{4\pi T} - \frac{D}{4\pi T}\frac{\partial^2}{\partial\mathbf{r}^2}\right)\right|\Delta_1 = $$

$$= \frac{i}{\pi T}\psi'\left(1/2 + \frac{\Gamma_\infty}{2\pi T}\right)\left[eD\mathbf{A}_1\frac{\partial\Delta}{\partial\mathbf{r}} + \frac{eD}{2}\Delta div\mathbf{A}_1\right] + $$

(10)

$$+\pi T\sum_{\omega_n>0}\Delta_1(\omega_n + \Gamma_\infty)^{-4}\left(\omega_n - \frac{D}{2}\frac{\partial^2}{\partial\mathbf{r}^2}\right)\Delta^2 + I(\omega)$$

$$I(\omega) = -\int d\varepsilon\left[th\left(\frac{\varepsilon+\omega}{2T}\right) - th\left(\frac{\varepsilon}{2T}\right)\right]W(\varepsilon)$$

$$\left[-i\omega - D\frac{\partial^2}{\partial\mathbf{r}^2} + \Delta^2\left(\frac{1}{-i\varepsilon + \Gamma_\infty} + \frac{1}{i\varepsilon + \Gamma_\infty}\right)\right]^{-1}\left[e(\varphi + Ddiv\mathbf{A}_1) + \right.$$

(10')

$$\left. +i\frac{\Delta\Delta_1}{2}\left(\frac{1}{-i\varepsilon + \Gamma_\infty} + \frac{1}{i\varepsilon + \Gamma_\infty}\right) - \frac{eD}{4}A_1\frac{\partial\Delta^2}{\partial\mathbf{r}}\left(\frac{1}{-i\varepsilon + \Gamma_\infty} - \frac{1}{i\varepsilon + \Gamma_\infty}\right)^2\right]$$

where

$$W(\varepsilon) = 2\Gamma_\infty\Delta\left(\varepsilon^2 + \Gamma_\infty^2\right)^{-1}$$

(10'')

Eqs.(8) - (10) along with the Maxwell equation:

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$$rotrot\mathbf{A}_1 + \left(\partial^2 \mathbf{A}_1 / \partial t^2\right) + \left(\partial / \partial t\right)\partial\varphi / \partial\mathbf{r} = 4\pi j \qquad (11)$$

form a general total system of equations determining the a.c. response for an inhomogeneous superconducting system.

B. Superconducting regions in a normal matrix

Let us now focus on the important case of inhomogeneous system when the normal metal contains a set of superconducting "islands". In the region T_C res.$<T< T_C$ * the order parameter is different from zero inside of the "islands" where the concentration of magnetic impurities and, correspondingly ,the value $\Gamma(\mathbf{r})$, is smaller than in the normal matrix :

$$\Gamma(\mathbf{r}) = \Gamma_\infty - \delta\Gamma(\mathbf{r}) \ , \delta\Gamma > 0 \qquad (12)$$

Let us denote $\tilde{\Delta}$ the localized state which exists in the potential $\delta\Gamma(\mathbf{r})$ (cf.[1]); $\tilde{\mu}$ is the lowest eigenvalue and $\tilde{\Delta}$ is the solution of the equation:

$$\left[-\delta\Gamma(\mathbf{r}) - (D/2)\left(\partial^2 / \partial\mathbf{r}^2\right)\right]\tilde{\Delta} = -\tilde{\mu}\tilde{\Delta} \qquad (13)$$

The solution of the equation (1''') for the order parameter can be seeked in the form $\Delta = B\tilde{\Delta}$, where $\tilde{\Delta}$ is the normalized eigenfunction for Eq.(13). With the use of Eqs. (1'''), (13), we obtain the following expression for the parameter $B \equiv B(T)$ which determines the temperature dependence of the order parameter and, correspondingly, the temperature dependence of the impedance Z (see below, Eqs. (26)- (28)).

$$B^2 = (4\pi T)^2 \left[\ln(T_C^* / T) + \psi\left(0.5 + \gamma^*\right) - \psi\left(0.5 + \gamma\right) \right] \times$$
$$\times \left\{ (D/12\pi T) v \psi'''(0.5 + \gamma) - u\left(\psi''(0.5 + \gamma) + \right.\right. \tag{14}$$
$$\left.\left. + (\gamma/3) \psi'''(0.5 + \gamma) \right) \right\}^{-1}$$

$$\gamma \equiv \gamma(T) = (\Gamma_\infty - \tilde{\mu}) / 2\pi T; \quad \gamma^* = \gamma\left(T_C^*\right)$$

Here
$$u = \int d\mathbf{r}\, \tilde{\Delta}^4, \quad v = \int d\mathbf{r}\left(\partial \tilde{\Delta}^2 / \partial \mathbf{r}\right)^2;$$

the expression for u and *v* see in the Appendix.

Eq. (14) is valid if $\Delta \ll \Gamma + \pi T$. Therefore, for $a \cong \xi$, Eq. (14) holds for a whole temperature range: $T_c^{res} < T < T_c^*$.

Near T$_C$* we obtain

$$B = \tilde{B}\left(1 - T / T_C^*\right)^{1/2} \tag{15}$$

Expression for $\tilde{B}$ can be obtained directly from Eqs.(14) and (A.6), see fig.2.

III. **a.c. Response of the Inhomogeneous Superconductor. Surface Impedance**

A. **Order parameter and a.c. conductivity of inhomogeneous layered system.**

Based on Eqs. (8) - (11) one can evaluate the a.c. response of the inhomogeneous superconductor. We focus on the most interesting case of layered systems. The results for the usual 3D case will be discussed in Section IV.

Our goal is to calculate the a.c. current for the inhomogeneous system, see fig. 1. Let us start with Eq. (10). The solution of this equation can be seeked in the form:

$$\Delta_1 = \Delta_1^o + \tilde{\Delta}_1 \tag{16}$$

where

$$\Delta_1^o = 2ie\sum_i \left(\mathbf{A}_1(\mathbf{r} - \mathbf{r}_j)\right)\Delta_j \; ; \tag{17}$$

The summation is taken over all "islands" j, Δ_j is the order parameter of the j-th island in the absence of the field; $\tilde{\Delta}_1 \ll \Delta_1^o$. Substituting Eqs. (16), (17) into Eq. (10) we obtain, after some manipulations, the following expression for the average value of the a.c. current.

$$\langle j \rangle = i\sigma\omega\mathbf{A}_1 + (\sigma/\pi T)\psi'\left(0.5 + (\Gamma_\infty/2\pi T)\right)\times$$
$$\left\{ < \sum_{k,j}\left(\mathbf{A}_1(\mathbf{r}_k - \mathbf{r}_j)\right)\Delta_k(\partial\Delta_j/\partial\mathbf{r}) > + \right. \tag{18}$$
$$\left. + (i/e) < \tilde{\Delta}_1(\partial\Delta/\partial\mathbf{r}) > \right\}$$

With the use of Eqs.(13), (18), and div**j**=0 (see Appendix), we arrive at the following general expression for the current density:

$$\mathbf{j} = \sigma_{eff}.\mathbf{E} \tag{19}$$

where

$$\sigma_{eff} = \sigma\left\{1 + \frac{1}{\pi T A_1^2}\psi'\left(\frac{1}{2} + \frac{\Gamma_\infty}{2\pi T}\right)\left[-\frac{i}{\omega}\sum_{k,j} < \left(\mathbf{A}_1(\mathbf{r}_k - \mathbf{r}_j)\right)\Delta_k\left(\mathbf{A}_1\frac{\partial\Delta_j}{\partial\mathbf{r}}\right)\right] > - \right.$$
$$\left. -\frac{(4\pi\sigma D - 1)}{D}\sum_j < \left(\mathbf{A}_1(\mathbf{r} - \mathbf{r}_j)\right)^2\Delta_j^2 > \right\} \tag{20}$$

σ is the d.c. conductivity. We assume an isotopic distribution of the "islands". The first term in the square brackets describes the overlap between different "islands". This term contributes to the imaginary part of the conductivity and, correspondingly, to a difference between the real and imaginary parts of the surface impedance (see below).

The second term contains contribution of separate "islands" and contains their order parameters. This term determines the temperature dependence of the impedance. It essential to note that the imaginary part in Eq.(20) contains the ω^{-1} dependence. The microwave region which is our main focus is characterized by low frequency , and the presence of such factor leads to a large increase in the imaginary term.

Assume that the vector potential is directed along the layers. The normalized solution of Eq.(13) is:

$$\tilde{\Delta} = C \ J_0(\kappa\rho) \quad ; \qquad \rho < \rho_o$$

$$\tilde{\Delta} = C \ J_0(\kappa\rho_0)\frac{K_0(t\rho)}{K_0(t\rho_0)} \quad ; \quad \rho > \rho_o \tag{21}$$

where ρ_o is the radius of the circle ("island"), J_0, K_0 are the Bessel functions, and

$$C = (2\pi)^{-1/2} \ \varphi_1^{-1/2} \tag{22}$$

$$\kappa = \left[2(\delta\Gamma - \tilde{\mu})/D\right]^{1/2}; \quad t = (2\tilde{\mu}/D)^{1/2}, and$$

$$\varphi_n = \int_0^{\rho_0} d\rho\rho^n J_0^2(\kappa\rho) + (J_0(\kappa\rho_0)/ K_0(t\rho_0))^2 \int_{\rho_0}^{\infty} d\rho\rho^n K_0^2(t\rho) \tag{23}$$

The lowest eigenvalue $\tilde{\mu}$ can be found from Eq.(13) and is determined by the equation:

$$J_1(\kappa\rho_0)/ J_0(\kappa\rho_0) = (t/\kappa)K_1(t\rho_0)/ K_0(t\rho_0) \tag{24}$$

It is interesting that for the 2D case there will be always a solution even with small exponential value of $\tilde{\mu}$ (cf. [26],p. 163); it has a form:

$$\tilde{\mu} = \left(\frac{2D}{\gamma^2 \rho_0^2}\right) \exp\left(-2\sqrt{D/\delta\Gamma}/\rho_0\right)$$

where $\gamma \cong 0.58$ is the Euler constant

With use of Eqs. (15), (20), and (21) we obtain the following expression for the a.c. conductivity:

$$\sigma_{eff} = \sigma_1 + i\sigma_2$$

$$\sigma_1 = \sigma\left\{1 + \left[(1 - 4\pi\sigma D)/DTV\right]\Psi'\left(0.5 + \Gamma_\infty/2\pi T\right)\sum_j B^2(T)C^2\varphi_3\right\}$$

$$\sigma_2 = (\sigma/2\pi TV\omega)\Psi'\left(0.5 + \Gamma_\infty/2\pi T\right)\sum_{k,j} I_{kj} \tag{25}$$

$$I_{kj} = \left(\pi^{3/2}/2\right)\left(J_0(\kappa\rho_0)/K_0(t\rho_0)\right)^2 B^2 C^2 \left(D/2\tilde{\mu}\right)^{1/4} \times$$

$$\times \left|r_k - r_j\right|^{3/2} \exp\left(-t\left|r_k - r_j\right|\right)$$

where κ, t and C are defined by Eqs. (22), (23), V is the volume of the system; the temperature dependence of the conductivity is determined by the factor B(T) (see Eq.(14)and fig. 2). It is essential that Eq.(25) contains only experimentally measured parameters and can be used for an analysis of experimental data (see below).

Eq.(25) allows to calculate the surface impedance Z, since

$$Z = \left(\frac{\omega}{4\pi\sigma}\right)^{1/2} \exp(-i\pi/4) \tag{26}$$

(see, e.g. , [27]). One can see directly from (25), (26) that, contrary to the usual normal metal, the metallic compound which contains superconducting "islands", is characterized by strong inequality: ReZ $\neq$ |Im Z|. Indeed, for normal metals the difference is negligibly small and connected with dependence: $\sigma(\omega) = \sigma_0(1 - i\omega\tau_{tr})^{-1}$; in our case $\omega\tau_{tr} \ll 1$.

In the temperature region close to T_C^*, i.e. $(T-T_C^*) \ll T_C^*$, the expressions for the conductivity σ_{eff} and, correspondingly, for the impedance, can be simplified and we obtain:

$$\operatorname{Re} Z = \tilde{Z}_n\left[1 - (\sigma_2/2\sigma_1)\right]$$
$$\operatorname{Im} Z = -\tilde{Z}_n\left[1 + (\sigma_2/2\sigma_1)\right] \tag{27}$$

Here

$$\sigma_2/2\sigma_1 = (s/\omega)(T_C^* - T)$$

(27')

or

$$\operatorname{Re} Z = -\operatorname{Im} Z - \omega^{-1}\left(2s\tilde{Z}\right)\left(T_c^* - T\right) \tag{28}$$

Here $\tilde{Z}_n = (\omega/8\pi\sigma_1)^{1/2}$. The expression for the parameter s can be obtained directly from (25), see below, Eq. (32). Near T_C^* the concentration of superconducting "islands", n_S, is small, and the sum in Eq.(25) contains contribution of nearest neighbors only. As a result, the quantity s depends exponentially on n_S.

Note that ReZ = |ImZ| at $T \geq T_C^*$. This can be seen directly from (27), (28). The inequality ReZ ≠ |ImZ| at $T \leq T_C^*$ is caused by the presence of superconducting "islands" and is described by the second term in Eq.(28). It is essential that this term is proportional to $\omega^{-1/2}$. Small value of the frequency ω in the microwave region leads this term to make a noticeable contribution. In addition, the dependence $\propto \omega^{-1/2}$ can be directly measured experimentally (see discussion below, Sec. V).

B.   Stripe-line "islands"

Consider different geometry of the "islands", namely the case when they have the stripe-line shape with a width "a". Then one can see that the impedance has a similar frequency and temperature dependences, although the numerical factors are different. Indeed, if the dependence $\Gamma(\mathbf{r})$ is such that its change is equal to $\delta\Gamma$ inside the "island" (cf. Eq.(12)), when the solution of the equation (13) is

$$\tilde{\Delta} = A \; \cos(\kappa x) \text{ for } |x| < a/2$$

and

$$\tilde{\Delta} = A \; \cos(\kappa a/2)\exp(ta/2)\exp(-t|x|) \text{ for } |x| > a/2$$

(the parameters κ and t are defined by Eq. (23); we will not write out an explicit expression for the normalization constant A). The eigenvalue $\tilde{\mu}$ is a solution of the equation:

$$tg(\kappa a/2) = \left(\tilde{\mu}/(\delta\Gamma - \tilde{\mu})\right)^{1/2} \tag{29}$$

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The value of $\tilde{\mu}$ is not equal to zero for any value of a (cf. [26]) and increases quickly with increase in a. Indeed $\tilde{\mu} = a^2 \delta \Gamma / 2D$ for small a. Based on Eq. (20), we obtain, after calculations the following expression (cf. [25]):

$$
\begin{aligned}
\sigma_{eff} = \sigma \Big\{ 1 + B^2 \psi^{'} (0.5 + \Gamma_\infty / 2\pi T) / 2\pi T V \times \\
\times \Big[(i/\omega) \sum_{kj} (d_{kj} / t) \cos\varphi_{kj} C^2 \cos^2(\kappa a / 2) \exp(-t d_{kj}) \times \\
\times \sin^{-1}\varphi_{kj} (1 - \cos\varphi_{kj})^{-1} ((1 - \exp(-b_j t \sin\varphi_{kj} / 2)) - \\
- \exp(-t d_{kj} \cos\varphi_{kj} (1 - \cos\varphi_{kj}))(1 - \exp(-b_j t \sin\varphi_{kj} x / 2) x^{-1}) + \\
(1 - 4\pi\sigma D)(12D)^{-1}) \sum_j b_j^3 \Big] \Big\}
\end{aligned}
\tag{30}
$$

where $x = \sin^2 \varphi_{kj} + \cos\varphi_{kj}$.

The main contribution to the sum in (30) comes from small angles φ_{kj}. The averaging over angles (we assume an isotropic distribution of stripes) leads to the result:

$$
\begin{aligned}
\sigma_{eff} = \sigma \Big\{ 1 + B^2 \psi^{'} (0.5 + \Gamma_\infty / 2\pi T) / 2\pi T V \times \\
\times \Big[(i/\pi\omega) C^2 \cos^2(\kappa a / 2) \sum_{kj} d_{kj} (d_{kj} + t^{-1}) \exp(-t d_{kj}) \ln(b_j t) + \\
+ ((1 - 4\pi\sigma D) / 12D) \sum_j b_j^3 \Big] \Big\}
\end{aligned}
\tag{30'}
$$

minimum distance between the stripes, and φ_{kj} is the angle between their axis.

IV. **Inhomogeneous distribution of coupling constants. 3D case.**

A. Inhomogeneous coupling constant; a.c. responce

As was noted above, the inhomogeneity of the superconductor can be caused by a non-uniform distribution of pair-breakers or by inhomogeneity

of the coupling constant $\lambda \equiv \lambda(r)$. [1]. Above (Secs. II, III) we focus on the inhomogeneity of the pair-breakers, so that the superconducting "islands" contain relative small number of such scatters and, as a result, characterized by larger value of T_C. Consider now the case of non-uniform distribution of the coupling constant. One can show that the a.c. response in this case is described by relations similar to those obtained above with some modification of the parameters.

The dependence $|\lambda(r)|^{-1}$ can be written in the form:

$$|\lambda(r)|^{-1} = |\lambda^o|^{-1} - \Lambda(r) \tag{31}$$

where the first term on the right side corresponds to the average value of the coupling constant. Consider a linear equation which follows directly from Eq. (2''') with $\lambda \equiv \lambda(r)$:

$$\left(|\lambda^o|^{-1} - \Lambda(r)\right)\tilde{\Delta} - 2\pi T \sum_{\omega_n > 0} \hat{L}^{-1}\tilde{\Delta} = -\tilde{\tilde{\mu}}\tilde{\Delta} \tag{31'}$$

$\hat{L}$ is defined by Eq. (3).

By an analogy with previous treatment, the solution of Eq. (2''') can be seeked in the form $\Delta = B_\lambda \tilde{\Delta}$. We will not write out the explicit expression for B_λ.

If Λ is small, its contribution can be treated as a perturbation. Then the equation for $\tilde{\Delta}$ in a main approximation has a form:

$$\{\ln\left(T_C^o / T\right) - \left(\Psi(0.5 + (\Gamma / 2\pi T)) - \Psi(0.5)\right) +$$
$$+ \Lambda(r) + (D / 4\pi T)\Psi'(0.5 + (\Gamma / 2\pi T))\partial^2 / \partial r^2\}\tilde{\Delta} = \tilde{\tilde{\mu}}\tilde{\Delta}$$

As a result, we obtain the expressions for the conductivity σ similar to Eqs. (20), (25) with the replacement $\tilde{\mu} \to \tilde{\tilde{\mu}}$.

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B. 3D inhomogeneous system

We focused above on the layered inhomogeneous superconducting systems. This is, indeed, the most important case (see below, Sec. V). However, one should note that a similar "pseudogap" scenario might occur for 3D inhomogeneous system. Let us describe also this case. All general equations (Sec II) are applicable also for the 3D system. The general equation for the a.c. conductivity, Eq. (20) is also valid; it contains a sum over 3D superconducting "clusters". The normalized solution of Eq. (13) in this case has a form:

$$\tilde{\Delta} = \left(C_{3D}/\rho\right)\left(\delta\Gamma/\left(\delta\Gamma - \tilde{\mu}\right)\right)^{1/2}\exp\left(-t\rho_0\right)\sin(k\rho); \quad \rho < \rho_0$$
$$\tilde{\Delta} = \left(C_{3D}/\rho\right)\exp\left(-t\rho\right) \qquad\qquad\qquad \rho > \rho_0 \tag{32}$$

Here ρ_0 is the radius of the "cluster"; t and κ are defined by Eq. (23); we will not write out the explicit expression for the normalization constant C_{3D}.

In the 3D case there is a minimum value of $\delta\Gamma$ for an appearance of the eigenvalue, and consequently, for a formation of the superconducting "cluster". Namely, the condition $\left(2\delta\Gamma/D\right)^{1/2}\rho_0 \geq \pi/2$ should be satisfied.

Consider the region close to T_c^{*}, where the distance between the "clusters" is large, so that $\left|r_k - r_j\right| \gg \rho_0; \left(D/2\tilde{\mu}\right)^{1/2}$. Based on Eqs. (20), (32), one can calculate the a.c. conductivity. For the imaginary part σ_2 we obtain the expression (cf. (25))

$$\sigma_2 = \left(\sigma/3\pi TV\omega\right)\psi'\left(0.5 + \Gamma_\infty/2\pi T\right)\times$$
$$\times\left\{\left(D/2\tilde{\mu}\right)^{1/2} + \delta\Gamma\left(\delta\Gamma - \tilde{\mu}\right)^{-1}\left[\rho_o + \left(D\tilde{\mu}/2\right)^{1/2}/\delta\Gamma\right]\right\}^{-1}\times \tag{33}$$
$$\times\sum_{kj} I_{kj}^{3D}$$

where $I_{kj}^{3D} = B^2 |r_k - r_j| \exp\left[-t\left(|r_k - r_j| - 2\rho_0\right)\right]$.

Note that the impact of inhomogeneity of superconducting properties is manifested much stronger for the layered structure. We will discuss it in more detail below (see Sec. V). Qualitatevely, it is due to the fact that the proximity effect which depresses the superconducting state is manifested much stronger in the 3D case when the superconducting "cluster" is affected by the normal matrix in all spatial directions. The layering is more efficient for the "pseudogap" phenomenon.

V. **Discussion**

We have evaluated the a.c. response (see Eqs. (25) - (28)) of an inhomogeneous superconductor in the pseudogap region ($T_c^{res} < T < T_c^{*}$). This region is characterized by superconducting "islands" embedded in a normal metallic matrix.

The surface impedance is described by Eq.(27), (28). The strong inequality ReZ $\neq$ |ImZ| is caused by the presence of superconducting "islands" and is described by the second term in Eq. (28). The frequency dependence ($\propto \omega^{-1}$) of this term can be measured experimentally. It would be interesting to carry out such measurements to verify our approach. The quantity s depends on a number of experimentally accessible parameters, including geometry. For example, if we assume the values:

T_C^{*} = 200K, T_C^{res} = 110K; $\Gamma_\infty = 160K$,

$$\rho = 2.5\xi; \quad \xi = \left(D/T_C\right)^{1/2}, \quad n_S \cong 10^{-2}, \quad \omega = 2\pi 10^{10} \ s^{-1}, \tag{34}$$

we obtain:

$$s \cong 4 \bullet 10^8 s^{-1} K^{-1} \tag{35}$$

Measurements of Z for $HgBa_2Ca_2Cu_3O_{8-\delta}$ compound at T > T_C^{res} were performed in [28]. It has indeed been observed that the slopes of the temperature dependences are different meaning that ReZ ≠ ImZ. With the experimentally determined value of the slope, we can calculate the parameter s, and it turns out to be close to the value (35). This collaborates our choice of parameters.

We think that a detailed a.c. response study of such materials should be based on a combination of various properties. For example, one can employ microwave and tunneling measurements. Indeed, the absolute value of the real part of impedance, ReZ, can be measured directly. However, this is not the case for ImZ; only the slope of the ImZ can be measured. This is sufficient to obtain an essential information. Indeed, if the slopes for ReZ and ImZ are different, it can be concluded that ReZ ≠ |ImZ|. On the other side, the tunneling measurements allow one to determine T_C^*. By putting $Re\, Z \cong |Im\, Z|$ at T = T_C^*, and using the slope of ImZ (from microwave data), one can obtain complete description on the a.c. response. It would be interesting to perform a.c. measurements and tunneling spectroscopy on the same sample.

As mentioned above, the presence of superconducting regions embedded in normal matrix is manifested stronger in layered structure as compared to 3D systems. Indeed, for the same set of the parameters as for the 2D case (see Eq. (34)), one calculates that the same value of s requires the concentration of "clusters" to be on order of $5 \cdot 10^{-2}$, which is much higher

than $n_{2D} \cong 10^{-2}$ (see Eq. (34)). Therefore, layering leads to a much stronger effect.

As far as cuprates are concerned, it is beneficial to use samples with composition far from the optimum doping. Indeed, according to neutron data [15, 16], such compound possess intrinsic inhomogeneity which, according to tunneling data [3 - 5], correlates with the appearance of the pseudogap phenomenon.

Recent tunneling data [13] indicates that superconducting "islands" are present in the Na-doped WO_3 compound. It would be interesting to perform microwave measurements for these samples.

Recently studied Pb + Ag system [14] displays a combination of normal resistance and gap structure. We think that its a.c. response can also be described by the theory developed here. A more detailed analysis of this system and of Na + WO_3 will be described elsewhere.

VI. **Summary. Acknowledgments.**

An inhomogeneous system containing superconducting regions ("islands") embedded in a normal matrix is characterized by peculiar a.c. transport. Its d.c. conductivity is normal, but the presence of the "islands" results in the strong inequality ReZ ≠ |ImZ|, in sharp contrast with the situation in normal metals. The difference is caused by an additional term proportional $\omega^{-1/2}$ (Eq. (28)), and smallness of ω has a noticeable impact on

microwave properties. The analysis of the a.c. response described above is based on our approach described in [1, 2] and is related to the pseudogap phenomenon in the cuprates [3 - 12], and to the observed properties of Na + WO_3 [13] and Ag + Pb systems [14]. We hope that interesting experimental studies of the microwave properties of such systems will be performed in the near future.

Appendix

1. In the presence of an external a.c. field the Green's function is presented by Eq. (5). The correction caused by the a.c. in the "dirty" limit is a solution of the equation (see [25]):

$$-i\omega_+\hat{\sigma}_z\hat{G}_1 + i\omega_n\hat{G}_1\hat{\sigma}_z + e\varphi\left(\hat{G}_0(\omega_n) - \hat{G}_0(\omega_+)\right) - \hat{\Delta}_1\hat{G}_0(\omega_n) + \hat{G}_0(\omega_+)\hat{\Delta}_1 -$$

$$-i\Delta\left[\hat{\sigma}_y\hat{G}_1\right] = -eDA_1\left(\hat{\sigma}_z\hat{G}_0(\omega_n)\partial/\partial\mathbf{r}G_0(\omega_n) -\right.$$

$$-G_0(\omega_+)\partial/\partial\mathbf{r}G_0(\omega_+)\hat{\sigma}_z) - eD\partial/\partial\mathbf{r}\left(A_1G_0(\omega_+)\hat{\sigma}_zG_0(\omega_n)\right) - iD\partial/\partial\mathbf{r}\left(\hat{G}_1\partial/\partial\mathbf{r}\hat{G}_0(\omega_n) + G_0(\omega_n)\partial/\partial\mathbf{r}G_1\right)$$

$$+(i/2)\Gamma(\mathbf{r})\left[\hat{\sigma}_z,\hat{G}_1\hat{\sigma}_z\hat{G}_0(\omega_n) + \hat{G}_0(\omega_+)\hat{\sigma}_z\hat{G}_1\right] +$$

$$+eD\hat{\sigma}_z\,div\mathbf{A}_1 \tag{A.1}$$

with use of Eqs. (7'), (A.1) and performing an analytical continuation (see [24], [29]), we obtain:

$$j = i\sigma\omega A_1 - \sigma A_1\left(\Delta^2/\pi T\right)\psi'\left(0.5 + \Gamma_\infty/2\pi T\right) -$$

$$-(i\sigma/2e\pi T)\psi'\left(0.5 + \Gamma_\infty/2\pi T\right)\left[\Delta(\partial\Delta_1/\partial r) - \Delta_1(\partial\Delta/\partial r)\right] - \tag{A.2}$$

$$-(i\sigma/2e)I_1(\omega)$$

$I_1(\omega)$ is defined by Eq. (10') with the replacement $W(\varepsilon)\to\partial/\partial r$. One can

show also that in a main approximation $\Delta_2 = -\Delta_1$.

Indeed, consider two regions for the variable ω_n. In the region

I (sign($\omega_n + \omega_0$) = sign(ω_n)) the function f_1 is the solution of the equation

$$-i\left(\omega_+ + \omega_n\right)f_1 + ie\varphi\left(\beta_+ - \beta\right) + \Delta_1\left(\alpha_+ + \alpha\right) =$$

$$= ieDA_1\left(\alpha\partial\beta/\partial r + \alpha_+\partial\beta_+/\partial r\right) + ieD\partial/\partial r\{A_1\left(\alpha_+\beta +\right.$$

$$+\alpha\beta_+\right)\} - iD\partial/\partial r\left(\alpha_+\partial f_1/\partial r\right) + i\Gamma\left(\alpha_+ + \alpha\right)f_1 \tag{A.3}$$

The function g_1 is determined in this region by the relation

$$g_1 = i\left(\beta_+ - \beta\right)\left(\alpha_+ + \alpha\right)^{-1}f_1.$$

For the region II (sign($\omega_n + \omega_0$) = -sign(ω_n)), one can write in a similar

way the equation for the function g_1. As for the function f_1, it can be found as

$f_1 = i\left(\beta_+ + \beta\right)\left(\alpha_+ - \alpha\right)^{-1}g_1$. It is essential that in a whole region we have $g_1 = g_2$

and $f_1 = -f_2$. The last relations leads to the relation $\Delta_1 = -\Delta_2$.

Eq. (A.2) contains the potential φ which can be excluded with use of the condition (9). Based on Eq. (7'), (A.2) we arrive at the expression for the a.c. conductivity (8).

2. With the use of Eqs. (10), (13) we obtain

$$
\left[\hat{\mu} - \delta\Gamma - (D/2)\partial^2/\partial r^2 - i\omega/2\right]A_1 =
$$
$$
= -2ie\; DA_1(\partial\Delta/\partial r) - ieD(div A_1)\Delta -
$$
$$
-\left(2\pi T/\Psi'(0.5 + \Gamma_\infty/2\pi T)\right)I(\omega)
$$

(A.4)

where I(ω) is determined by Eq. (10').

The scalar potential φ can be excluded with use of the equation: div **j** = 0. As a result, we obtain:

$$
-eD\;\partial^2\varphi/\partial r^2 = (ie\omega/\pi T)\Psi'(0.5 + \Gamma_\infty/2\pi T)\sum_j \left(A_1(r - r_j)\right)\Delta_j^2
$$

(A.5)

Based on Eqs. (18), (A.4) and (A.5), we arrive for an isotopic distribution of the "islands" at Eq. (20).

3. Eq. (14) contains the quantities u and ν These parameters can be calculated for different cases. For example, for the "stripe" geometry (see Sec. 3B), one can obtain:

$$
u = \int dx \tilde{\Delta}^4 = (C^4/2)\left[3a/4 + \kappa^{-1}\sin\kappa a +\right.
$$
$$
\left.+(1/8)\kappa^{-1}\sin 2\kappa a + t^{-1}\cos^4(0.5\kappa a)\right]
$$
$$
v = \int dx\left(\partial\tilde{\Delta}^2/\partial r\right)^2 = C^4\left\{2t\cos^4(\kappa a/2) +\right.
$$
$$
\left.+(\kappa/2)\left[\kappa a - 0.5\sin(2\kappa a)\right]\right\}
$$

(A.6)

The quantities C, κ, t are defined by Eqs. (22), (23).

Based on Eqs. (13), (21), one can calculate these quantities for the 2D "islands". For example, if we consider the case with parameters, (see Eq. (34)), we obtain $\upsilon = 0.06\xi^{-2}$, $v = 9\cdot10^{-3}\xi^{-4}$.

References

1.   Yu.Ovchinnikov, S.Wolf, V.Kresin, Phys.Rev.B63, 064524 (2001)

2.   Yu.Ovchinnikov, S.Wolf, V.Kresin, Phys.Rev.B60, 4329 (1999)

3.   Ch.Renner, B.Revaz,J-Y.Genoud, K.Kadowaki, O.Fischer ,
        Phys. Rev. Lett. 80, 149 (1998)

4.   M.Suzuki, T.Watanabe, A.Matsuda, Phys. Rev. Lett. 82, 5361 (1999)

5.   T.Ekino Y.Seraki, S.Mashimoto,H.Fujii, J.of Low Temp. Phys. 117, 359
    (1999)

6.   H.Ding, T.Yokoya, J.C.Campuzano, T.Takahashi, M.Randeria,   M.Norman,
        T.Mochiku, K.Kadowaki, J. Giapintzakis , Nature 382, 51 (1996)

7.   A.Loeser, Z.-X.Shen, D.S Dessau, D.S.Marshall, C.H.Park, P.Fournier,
        A.Kapitulnik., Science 273, 325 (1996)

8.   J.Loram, K.Mirza, J.Wade, J.Cooper, W.Liang., Phys.C 235, 134 (1994);
        J.Tallon, J.Cooper, P. de Silva, G..Williams, J.Loram., Phys. Rev. Lett. 75,
        4114 (1995); b) J.Tallon, J.Loram, (Preprint)

9.   D.R.Temprano, J.Mesot, S.Janssen, K.Conder, A.Furrer, H.Mutka,
        K.Mueller, Phys. Rev. Lett. 84, 1990 (2000)

10.   C.Bergemann, A.Tyler, A.Mackenzie, J.Cooper, S.Julian, D.Farrell.,
        Phys. Rev B 57, 14387 (1998)

11.   P.Caretta, A.Lascialfari, A.Rigamonti, A.Rosso, A.Varlamov., Phys. Rev. B
        61, 12420 (2000)

12.   I.Igushi, T.Yamagreshi, A.Sugimoto, Nature 412, 420(2001); we are
        grateful to Prof. I.Igushi for sending the preprint prior to publication.

13.   S.Reich and Y.Tsabba, Eur.Phys.J. B12, 13 (1999);
        A.Shengelaya, S.Reich, Y.Tsabba, and K.Mueller, Europhys. J. B12, 13
        (1999); S.Reich, G.Leitus, Y.Tssaba, Y.Levi, A.Sharoni, O.Millo.,

14.   L.Merchant, J.Ostrick, B.Barber, Jr., R.Dynes, Phys.Rev. B63, 134508
        (2001)

15.   a) T.Egami and S.J.L.Billinge, in Physical Properties of High Temperature
    Superconductors V.D.Ginzburg, Ed., p. 265, World   (Singapore), 1996;
                                            M. Gutmann, S.J.L.
    Billinge, E.L.Brosha, G.H.Kwei., Phys. Rev. B61, 11762   (2000)

16.   E. Bozin, G.Kwei, H.Takagi, S.Billinge., Phys. Rev. Lett. 84, 5856 (2000)

17.   V.Madharan, K.Lang, E.Hudson, S.Pan, H.Eisaki, S.Uchida, J.Davis., Bull. APS, 45,416(2000; K.Lang, V.Madhavan, J.Hoffman, E.Hudson, S.Pan, H.Eisaki, S.V Chida, J.C.Davis (Preprint)

18.   M.Tinkham, Introduction to Superconductivity, McGraw-Hill, NY (1996)

19.   G.Eilenberger, Z. Phys. 214, 195 (1968)

20.   A.Larkin and Yu.Ovchinnikov, JETP 28, 1200 (1969); A.Larkin and Yu.Ovchinnikov, in Non-Equilibrium Superconductivity, ed. by D.Landenberg and A.Larkin (Elsevier, Amsterdam, 1986), p.530

21.   A.Larkin and Yu.Ovchinnikov, JETP 34, 1144 (1972)

22.   L.Gor'kov, Sov.Phys., JETP 9, 1364 (1959)

23.   A.Abrikosov and L.Gor'kov, JETP 12, 1243 (1961)

24.   A.Abrikosov, L.Gor'kov, I.Dzyaloshinski, Methods of Quantum Filed Theory in Statistical Physics, Dover, NY (1963)

25.   A.Larkin, Yu.Ovchinnikov, J. of Low Temp. Phys. 10, 407 (1973)

26. L.Landau and E.Lifshits, Quantum Mechanics, Pergamon press, Oxford (1977)

27.   L.Landau and E.Lifshits, Electrodynamics of Continuous Media, Addison-Wesley, Reading, Mass. (1960)

28.   C.Kusko, Z.Zhai, N.Hakim, R.Markiewicz, S.Sridhar, D.Colsom, V.Viallet-Guillen, Yu.Netyadov, M.Trunin, N.Kolesnikov, A.Maignan, A.Erb (Preprint)

29.   Yu.Ovchinnikov, Sov.Phys. JETP 45, 404 (1977)